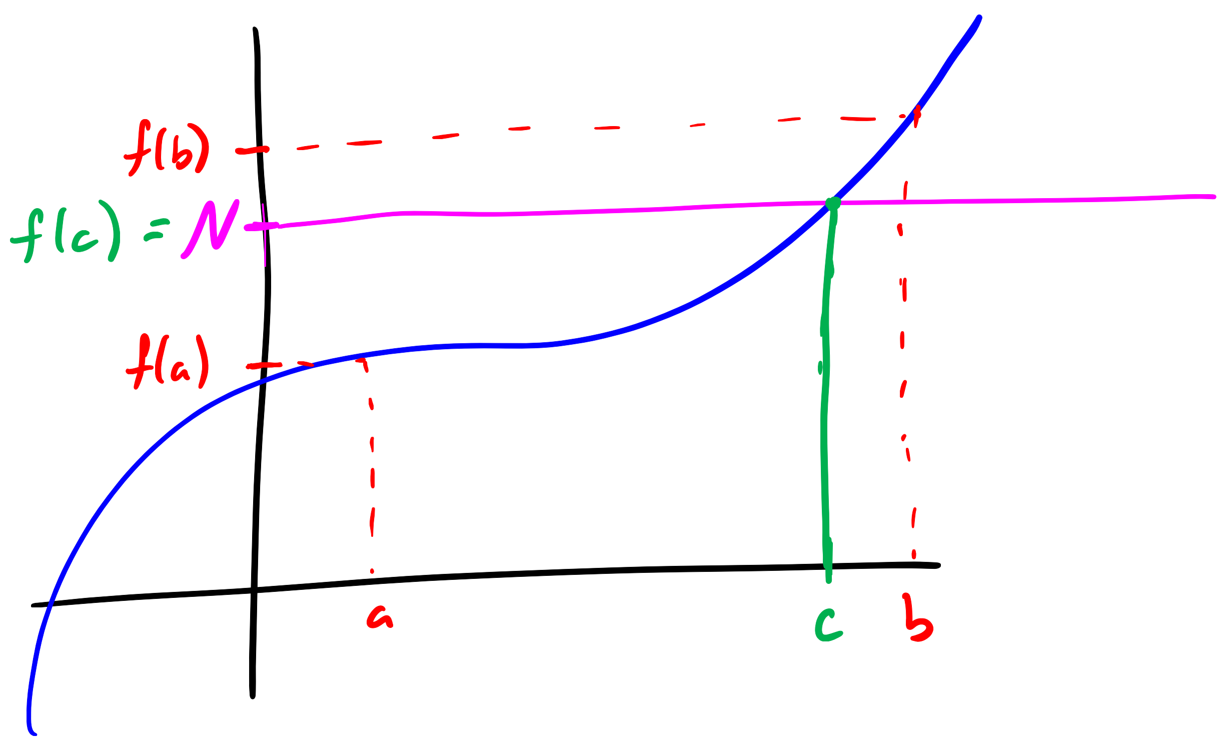
Intermediate Value Theorem

Suppose that f is continuous on the closed interval $[a,b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there is a number c in the interval (a,b) such that $f(c) = N$.



Ex: Show that $x^4 + x - 3 = 0$ has a solution on the interval $(1,2)$.

Sol: $f(x) = x^4 + x - 3$ is continuous on $[1,2]$ b/c it's a polynomial and

$f(1) = 1 + 1 - 3 = -1$ & $f(2) = 16 + 2 - 3 = 15$.

Since $f(1) < 0 < f(2)$, by the intermediate value theorem there is a number c in $(1,2)$ with $f(c)=0$. //

$$f(x) = \frac{x^2-1}{x^2+1}$$

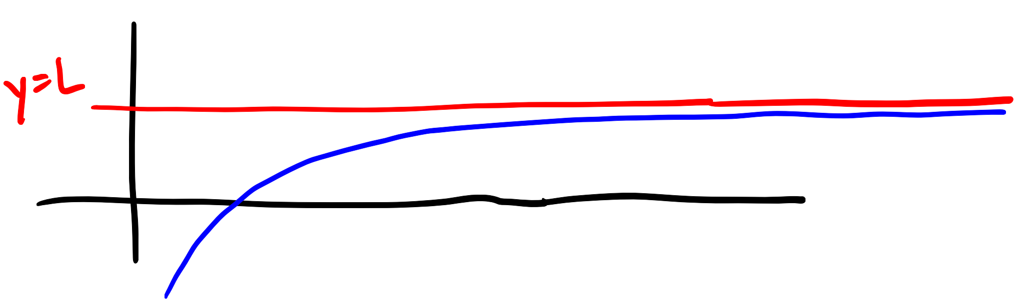
$$\lim_{x\to\infty} \frac{x^2-1}{x^2+1} = 1 \quad \& \quad \lim_{x\to-\infty} \frac{x^2-1}{x^2+1} = 1$$

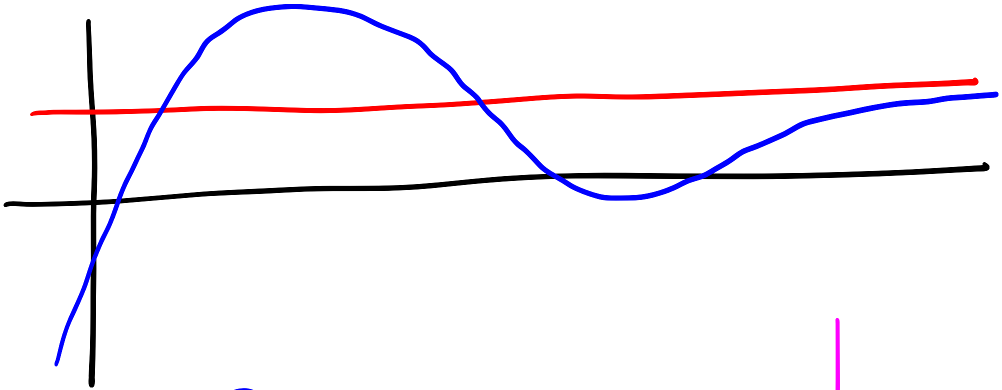
$y=f(x)$ has a horizontal asymptote at $y=1$.

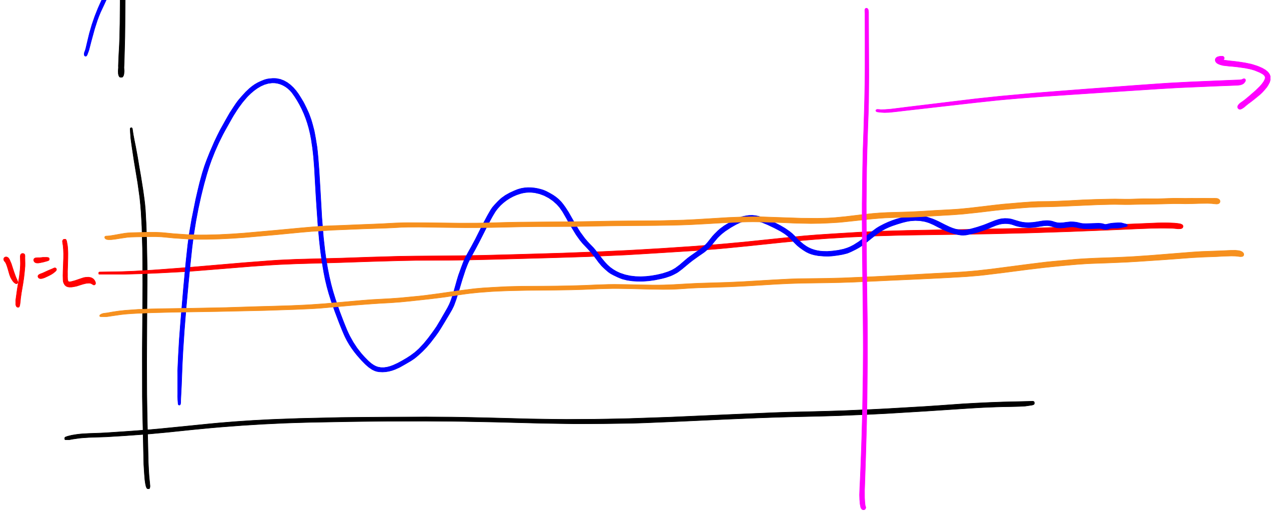
<u>Def</u>: Let f be a function defined on an interval (a,∞). Then

$$\lim_{x\to\infty} f(x) = L$$

means that the values of $f(x)$ approach L as x becomes arbitrarily large.







$$\lim_{x\to\infty} \frac{1}{x} = 0 \qquad \lim_{x\to-\infty} \frac{1}{x} = 0$$

$\downarrow$

If $r>0$ is a rational number, then

$$\lim_{x\to\infty} \frac{1}{x^r} = 0$$

If $r>0$ is a rational number such that x^r is defined for all x, then

$$\lim_{x\to-\infty} \frac{1}{x^r} = 0$$

Ex:

① $$\lim_{x\to\infty} \frac{4x^3+6x^2-2}{2x^3-4x+5} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}}$$

$$= \lim_{x\to\infty} \frac{4+\frac{6}{x}-\frac{2}{x^3}}{2-\frac{4}{x^2}+\frac{5}{x^3}} = \frac{4+0-0}{2-0+0} = 2$$

② $$\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+1}}$$